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Theory and Application of Coupling Between Curved Transmission Lines

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***Abstract*—An analytical method for deriving the fields, the reflection, and the directivity of two coupled curved transmission lines is described. The fields on both lines are found to be accurately in quadrature. The directivity and reflection are very small. The accuracy of the theoretical results for a 3 dB dielectric line coupler (designed at 94 GHz) is confirmed by experiment. Well-balanced outputs and a directivity of better than 40 dB are obtained. Though a substantial amount of insertion loss in the experimental model is found, this loss is believed to be largely dielectric loss. Design and performance data are presented.**

I. Introduction

IN THE PAST few years coupled-wave theory has become increasingly important due to its role in the millimeter-wave devices and integrated circuits. Coupled-wave theory between parallel transmission lines was originally introduced by Miller in a widely known paper [1]. Later, the problem of coupling between two parallel dielectric waveguides and its application to directional couplers has been studied extensively and numerous papers have been published [2]–[5]. Recently, the theory of coupling between parallel transmission lines has been extended to describe the coupling between nonparallel transmission lines [6]–[10]. Examples of practical interest are the application of coupling between two curved transmission lines to the design of directional couplers, and in some other cases, the need to avoid crosstalk [11], [12]. Although this case has received little attention, up to date, except for some recent work, such analysis is important due to the increasing interest in millimeter-wave integrated devices.

In a previous paper, [9] the authors have presented the basic formulation for the coupling between curved transmission lines, and considered the coupling problem of a finite length parallel coupler joined to matched loads via nonparallel transmission lines. Closed-form expressions for the field amplitudes, directivity, and reflection were also given. In the present paper, the field amplitude differential equations that were derived in the previous paper for the curved sections are used. By combining the WKB method and a perturbation technique, the differential equations are solved, and closed form expressions for the field amplitudes, the directivity, and the reflection coefficient (due to back-coupling) are derived. An experimental model for a 3-dB directional coupler designed at 94 GHz is constructed and tested. The experimental data are in good agreement

Manuscript received February 9, 1982; revised June 11, 1982.
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with the theoretical results though a substantial amount of insertion loss is found; this loss is believed to be largely dielectric loss.

II. Analysis

Fig. 1 illustrates the geometry used in the description of two nonparallel transmission lines. As shown, the structure is symmetrical and is composed of two continuously curved sections. The intention of not using a straight middle section is to avoid the reflection coming from the discontinuity between the straight and the curved parts, and to enable the maximum coupling to be increased, thus giving more bandwidth. A continuously curved section should exhibit reflections of a smaller amplitude. Fig. 1 shows a particular geometry for two curved transmission lines symmetrically located about a plane that contains the z-axis. In order to maintain radiation losses at a tolerable level, the minimum radius of curvature is assumed to be large compared to λ in the sense described by Lewin *et al.* [13]. Moreover, by assuming a large radius of curvature, the phase constants along both lines (which are taken identical) are, to the first order, equal to β_0, the phase constant of a straight transmission line [13]. As shown in Fig. 1, both lines are assumed to be of a parabolic form and are separated by a distance $d(z)$ given by

$$d(z) = d_0\left[1+\left(\frac{z}{L}\right)^2\right] \tag{1}$$

where d_0 represents the separation between the two lines at $z=0$, and L is the longitudinal distance from the center at which the separation distance is doubled. In this notation, d_0/L^2 corresponds to $2c$ in the previous paper [9], where c is the curvature parameter, and is half the inverse of the minimum radius of curvature R. Hence

$$L = \sqrt{Rd_0}. \tag{2}$$

In the previous paper, the transmission line equations which govern the actual coupled fields along each line were written. Upon making certain substitutions, the coupled transmission line equations were reduced to a set of decoupled second-order differential equations. As in any system involving coupled linear circuits, we have a set of simultaneous linear differential equations which admit a particularly simple set of solutions: the normal modes of the system (symmetrical and antisymmetrical modes) [14]. Extending the formulation to cover the coupling of modes on two curved lines results in complicating the differential equations, which now become

$$\frac{d^2I}{dz^2} + \beta_0^2 I = -2I\beta_0\Delta\beta f(z)$$

$$\frac{d^2i}{dz^2} + \beta_0^2 i = 2i\beta_0\Delta\beta f(z)$$

$$\frac{d^2V}{dz^2} + \beta_0^2 V = -2V\beta_0\Delta\beta f(z)$$

$$\frac{d^2v}{dz^2} + \beta_0^2 v = 2v\beta_0\Delta\beta f(z) \tag{3}$$

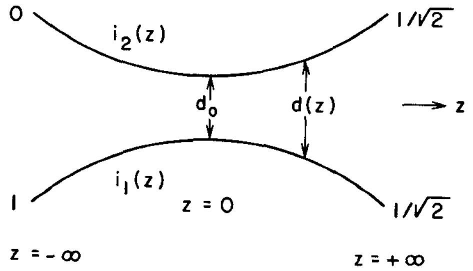

Fig. 1. Geometry and boundary conditions of the 3-dB coupler.

with

$$f(z) = e^{-hd_0(z/L)^2} \tag{4}$$

$$V = v_1 + v_2 \quad v = v_1 - v_2$$

$$I = i_1 + i_2 \quad i = i_1 - i_2. \tag{5}$$

Here, (I, V) and (i, v) stand, respectively, for the symmetrical and antisymmetrical modes, $i_{1,2}$ and $v_{1,2}$ are used to represent the coupled fields along each guide as shown in Fig. 1, β_0 is the phase constant (of the propagating mode) for a single line in isolation, $\Delta\beta$ is the shift in propagation constant due to coupling, and h is the decay constant of the evanescent field outside the line. The z-dependent exponential term that appears on the right-hand side of (3) is caused by the variable coupling along the curved lines of the coupler. As a result of the z-dependent term, there is no exact closed-form solution for these differential equations and, hence, an approximate analytical method will be employed. This z-dependent exponential term is small and it represents a small perturbation to the solution of the homogenous unperturbed differential equation. By combining the WKB method and a perturbation technique [15], an approximate solution for (3) is derived. The employment of the classical WKB method followed by the perturbation technique is found to give the known WKB solution (which represents a forward wave) augmented by an infinite number of correction terms. For the purpose of this study, only the first correction term need be retained. In fact, the complete solution appears to have been common knowledge for some time. The representation of the solution as a series appears first to have been given by Bremmer [16]. The method that was suggested by Bremmer has a very simple physical interpretation, based on the concept of wave propagation, refraction, and reflection. The basic principles underlying Bremmer's approach have been surveyed [17] and the question of its convergence was also considered [18]. A full analysis of the analytical method used here is given in the Appendix.

Assuming unit incident field $i_1(z)$ at $z = -\infty$ and matched terminations at $z = +\infty$, the expressions of the actual coupled field magnitudes, the reflection coefficient, and the directivity are given by

$$|i_1(\infty)| = \cos\left(\Delta\beta\sqrt{\frac{\pi R}{h}}\right) \tag{6}$$

$$|i_2(\infty)| = \sin\left(\Delta\beta\sqrt{\frac{\pi R}{h}}\right). \tag{7}$$

Reflection

$$|i_1(-\infty)| = 2(\Delta\beta)^2 \frac{\pi R}{h} e^{-\beta_0^2 R/h} + 0(\Delta\beta)^4. \quad (8)$$

Directivity

$$|i_2(-\infty)| = \Delta\beta\sqrt{\frac{\pi R}{h}}\, e^{-\beta_0^2 R/h} + 0(\Delta\beta)^3. \quad (9)$$

Moreover

$$\operatorname{Arg} i_2(\infty) - \operatorname{Arg} i_1(\infty) = -\pi/2. \quad (10)$$

From (10) we learn that a constant phase difference of 90° exists between the two guides. We also find that the phase of the guide in which the power is increasing will always lag 90° behind the phase of the guide in which the power is decreasing. Physically, the reason for this phase difference is the necessary phase relation between the dielectric polarization (caused by the field in guide 1), and the field in guide 2 if power is to be generated in guide 2 [9]. Equations (8) and (9) represent the expressions for the reflection coefficient and the directivity. It is clear from (8) that reflection is caused by the line curvature, and, hence, as R increases the reflection coefficient decreases. This fact can be seen from Fig. 2 where the curved line is represented by a finite number of linear sections connected to each other in tandem. For an incident wave, reflection will take place at each junction, and (8) represents the first term of a geometric optical series given by Bremmer [16].

Equation (9) is identified as the directivity of the coupler which comes from the reverse coupling from line (2) to line (1).

According to (6) and (7), the power will transfer back and forth between the two guides. The necessary condition for a complete transfer of power from one guide to the other is given by

$$\Delta\beta\sqrt{\frac{\pi R}{h}} = \frac{\pi}{2}. \quad (11)$$

In the case of equal power division, the amplitudes in the two lines become proportional to $1/\sqrt{2}$ and accordingly the design condition becomes

$$\Delta\beta\sqrt{\frac{\pi R}{h}} = \frac{\pi}{4} \quad (12)$$

where R is the minimum radius of curvature of the line, $\Delta\beta$ is the coupling coefficient, and h is the decay constant of the field outside the line. Clearly, for a given line, (12) determines the minimum radius of curvature of the line. For example, in the case of two dielectric waveguides separated by a distance d_0, the coupling coefficient is given by [5]

$$\Delta\beta = c_0 e^{-h d_0} \quad (13)$$

with

$$c_0 = \frac{hp^2}{\beta_0^2 k_0^2\left(\frac{a}{2} + 1/h\right)(\epsilon_r - 1)} \quad (14)$$

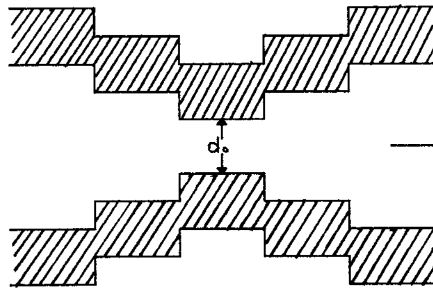

Fig. 2. Representation of two parallel curved lines as a finite number of parallel linear sections joined to each other in tandem.

where $k_0 = 2\pi/\lambda_0$, p is the transverse wave number inside the dielectric, and a is the width of the dielectric waveguide.

Substituting (13) into (12), we get the design equation

$$d_0 = \frac{1}{2h}\ln\left(\frac{16 c_0^2 R}{\pi h}\right). \quad (15)$$

By choosing suitable values for a, ϵ_r, R, and the frequency, the calculation of the separation distance d_0 becomes straightforward by using (15). Furthermore, the value of L can be evaluated by using (2). In fact, (15) describes the dependence of the separation distance d_0 on the radius of curvature R. That is, for the same lines, the separation distance d_0 decreases logarithmically as the curvature R decreases. In other words, tight coupling, which gives rise to a wider bandwidth, can be achieved by decreasing R. On the other hand, the radius of curvature R should be significantly large (about 12 λ_0) [13] so that radiation losses are negligible.

In the next sections, (13)–(15) will be used to evaluate the separation distance d_0 for a 3-dB dielectric directional coupler. A complete description of the experimental model and the experimental findings are also given.

III. Description of the Experimental Model

In order to test the accuracy of the theoretical results, a 3-dB dielectric directional coupler was constructed. This experimental model is composed of two teflon guides sandwiched between two parallel conducting plates. This type of dielectric line is usually called a double H-guide [20], [21]. The teflon guides are cut in the form of continuously curved strips and then placed between two parallel plates. Teflon is chosen for the waveguide material because of its expected low loss and ease of fabrication. The geometry of the described 3-dB dielectric coupler is shown in Fig. 3.

Transitions from metal to dielectric waveguides are made by using a tapered dielectric waveguide and an improved horn. In order to achieve a smooth transition, symmetric tapering is insured. The length of each tapered section of the dielectric guides is chosen to be about 3 λ_g. The tapered section of the dielectric guide is inserted into the waveguide section of the horn. On the basis of an earlier investigation by Trinh *et al.* [22], the flare angle θ of the

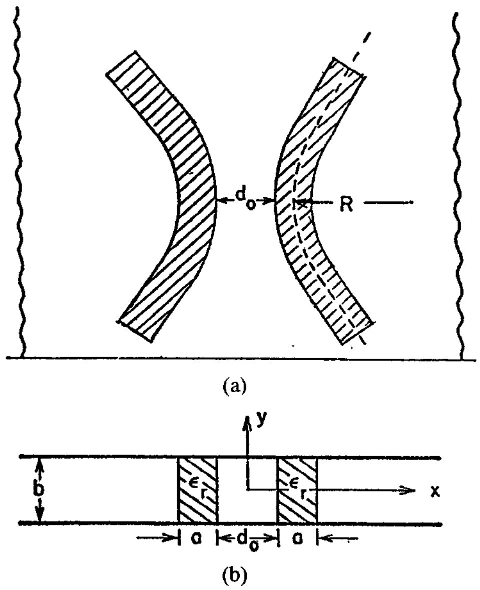

Fig. 3. (a) Top view of a 3-dB curved dielectric directional coupler. (b) Central cross section of the parallel plane waveguide partially filled with two curved dielectric guides and separated by a distance d.

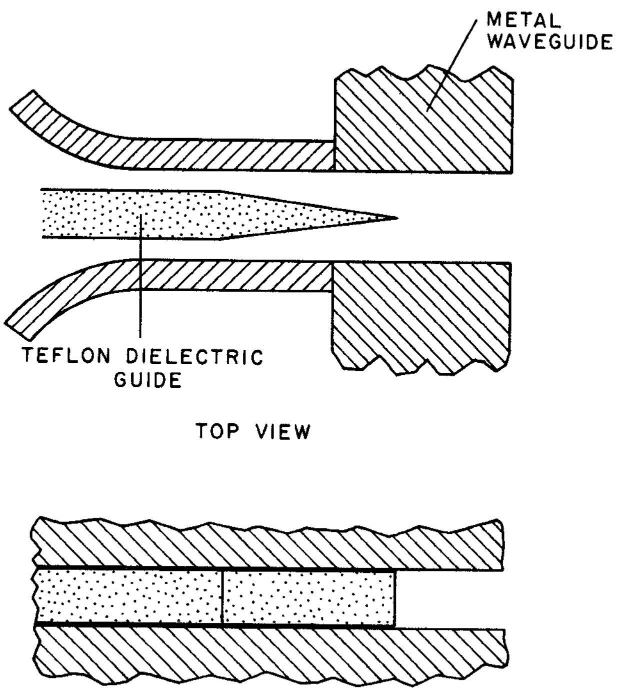

Fig. 4. Dielectric waveguide to metal waveguide transition.

horn is chosen to be in the H-plane and fixed at 35°. The length of the horn's variable width is 3 λ_g. The metal to dielectric transition is shown in Fig. 4.

In order to hold the dielectric waveguides in place, a low-loss adhesive (5-min Epoxy) is used. However, the measurements indicated the existence of a substantial amount of insertion loss which at the time was thought to be caused by the bonding material. In an attempt to eliminate these losses the adhesive was removed and some other mechanical supporting techniques were employed.

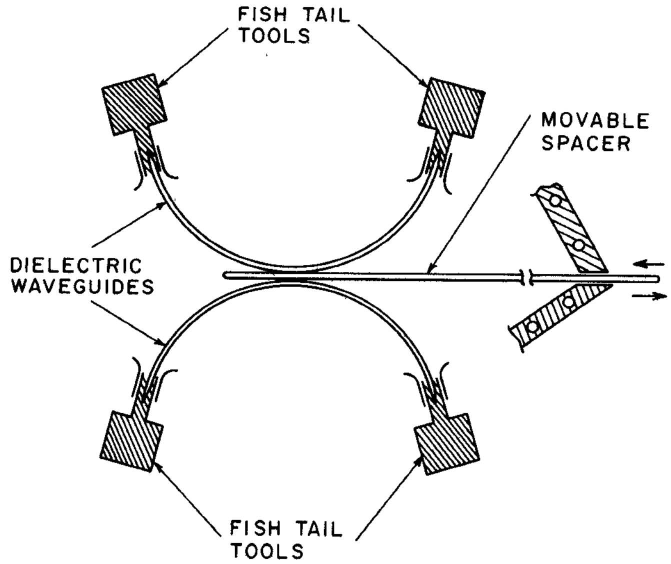

Fig. 5. Mechanical technique used to locate the dielectric waveguide in place.

The dielectric waveguides were held in place by means of movable spacer and fish-tail supports as shown in Fig. 5. The fish-tail support and the movable spacer were removed after fixing the top plate, which in turn held the two dielectric guides in place. Indeed, this mechanical technique turned out to be very reliable in the short run, even though it did not resolve the insertion loss problem as we will see in the next section.

The double H-guide shown in Fig. 3 is capable of supporting both TE and hybrid modes. An analytical description of the fields has been reported on by many authors [21], [23]. The fundamental mode of this system is the TE_0 even mode in which the electric field is vertically polarized. The second propagating mode is the first-order $H_x = 0$ hybrid mode. In this mode, the electric field is horizontally polarized and, hence, less conductor losses are expected [24]. However, feeding the coupler by the first $H_x = 0$ hybrid mode is found to give less coupling between the two guides and, hence, to give a relatively poor 3-dB bandwidth compared to the TE_0 even mode case [25].

IV. Experimental Results

In order to test the accuracy of the above theoretical results, a 3-dB dielectric directional coupler (designed at 94 GHz) was constructed and some measurements were made in the frequency-range of 94.25–95.5 GHz. A broad-band klystron was not available and, hence, the expected 20 GHz calculated bandwidth was not confirmable. The guiding medium was teflon with $\epsilon_r = 2.05$ and an *expected* $\tan\delta = 4\times10^{-4}$ (low frequency value). The guide dimensions were 1.35×1.27 mm, and the curved sections were separated by a minimum distance of 0.5 mm. The minimum bending radius of the curved guides was about 37 mm. Sheets of absorbing material (ECCOSORB) are inlaid inside the coupler (close to the conducting walls and at an appreciable distance from the dielectric guides) so that any spurious resonances are eliminated. The overall dimensions

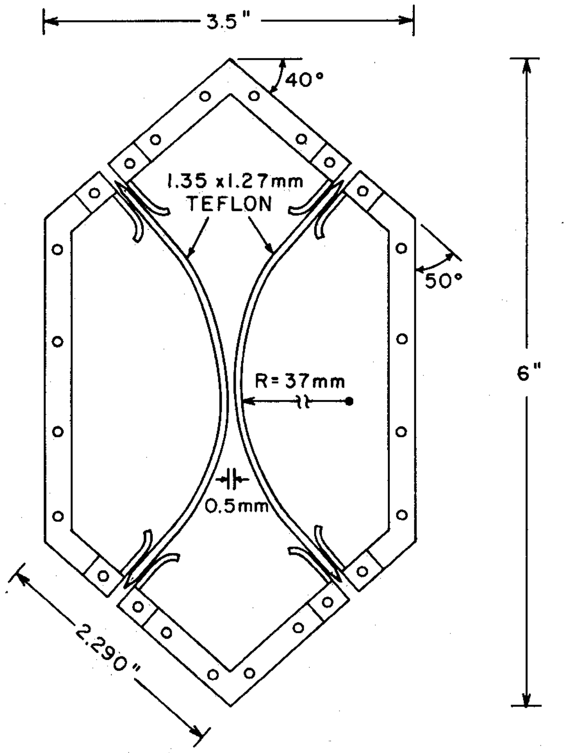

Fig. 6. The geometry of the measured 3-dB dielectric directional coupler.

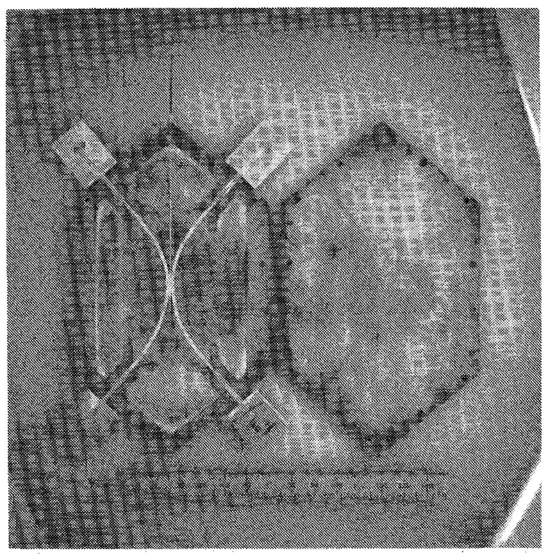

Fig. 7. Photograph of the constructed 3-dB coupler.

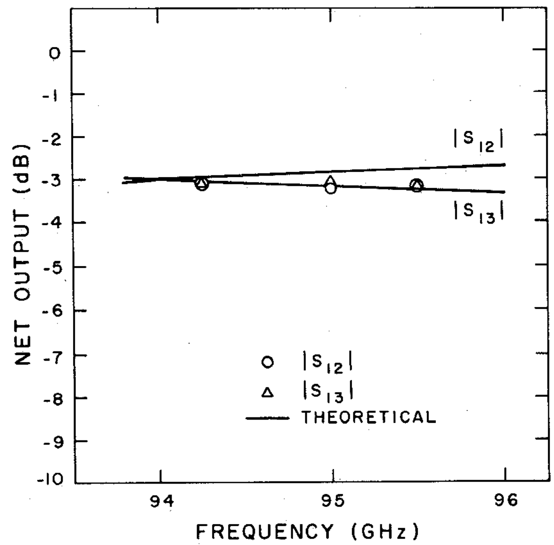

Fig. 8. Measured and theoretical outputs of the 3-dB coupler. (The measured values are corrected for material and coupling losses.)

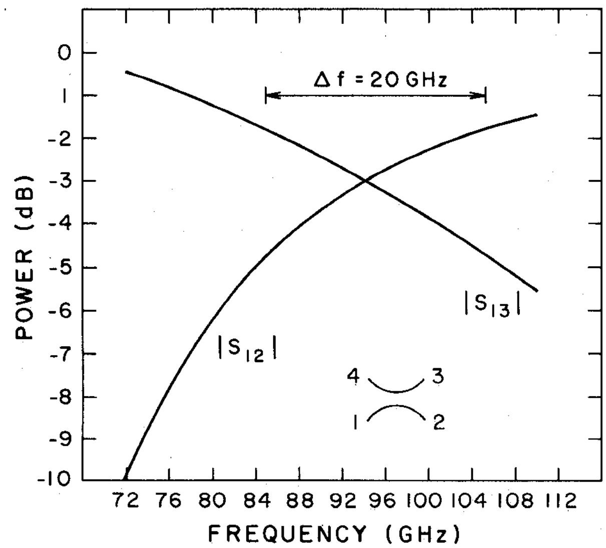

Fig. 9. Theoretical power/frequency characteristics of the proposed 3-dB dielectric directional coupler.

of the 3-dB coupler is 3.5in×0.75in×6in. A more detailed description of the model is shown in Figs. 6 and 7.

The reflection loss (or the conjugate mismatch loss) of the various input ports with matched terminations on the other ports was measured at 95 GHz and found to be less than 0.03 dB. The outputs of the coupler (for the case where adhesive was used) were measured at 95 GHz and found to be 8.5 dB below the input. The directivity and the reflection of the coupler were measured as at least 40 dB below the input. Because the outputs were well below the 3 dB level, it had been thought that this substantial amount of insertion loss could be caused in part by the adhesive. As a result, the bonding material was removed and an alternative means of mechanical support was introduced. A movable spacer and some fish-tail-like tools, as demonstrated in Fig. 5, were used to locate the dielectric prior to clamping between the plates.

Repeating the measurements at 95 GHz, the outputs of ports 2, 3, and 4 were 7.4, 7.4, and 40 dB below the input, respectively. Therefore, the replacement of the adhesive material reduces the insertion loss by only 1.1 dB. Theoretical calculations [21] indicate that the dielectric loss (for $\tan\delta = 4\times10^{-4}$) and the (ideal) conductor loss should be no more than 0.45 dB and 0.27 dB, respectively. The transition loss of the mode launcher could be of the order of 1.0–1.5 dB per transition pair [22], [26], figures which are consistent with our measurements.

In order to measure the dielectric loss, losses for two dielectric waveguides of different length were measured, and the loss of the shorter one was subtracted from that of the longer one. Thus, the loss of the two horn launchers was eliminated and the transmission loss only was obtained. The accuracy of the measurement was within 0.1 dB, and the transmission loss found was 2.8 dB per coupler length. Subtracting the transmission loss and the 1.5 dB horn launcher loss from the measured 7.4 dB gives a net output of 3.1 dB quite close to the design figure of 3.0 dB. The well-balanced performance of the coupler is shown in Fig. 8 for the frequency range 94.25–95.5 GHz. The theoretical power/frequency characteristics of the coupler for the frequency range 72–110 GHz is demonstrated in Fig. 9. The nearly equal power split is, if anything, better than the theoretical prediction.

In an attempt to confirm our dielectric loss measurements an extensive search in the literature was carried out. The ITT handbook [27] reported an increase in the value of $\tan\delta$ (for teflon) by a factor of 4 as frequency increases from 3 GHz to 25 GHz. Recently, W. Bridges *et al.* [28] reported the measurement of $\tan\delta$ for teflon at 94 GHz. These measurements indicate that the value of $\tan\delta$, for teflon, is 10 times larger than the low-frequency value. No specific explanations were given other than stating that, in general, dielectric losses are expected to be larger for higher frequencies due to lattice absorption. In addition, the authors tend to believe that this discrepancy might also be caused by measurement error or sample imperfections. Moreover, during the conduction of some measurements on a microstrip structure, F. Lalezari [29] detected a sharp increase in the dielectric loss as the frequency approached the millimeter region.

At any rate, based on our measurements, as well as the above mentioned references, we believe that a substantial amount of the insertion loss (about 3 dB) is due largely to a sharp increase of $\tan\delta$ with frequency in this band.

V. Conclusions

Using previously developed theory on coupling between curved transmission lines, approximate expressions for the amplitudes of the actual coupled fields, the reflection coefficient, and the directivity were derived. The fields on both lines are accurately in quadrature. The reflection coefficient and the directivity are exponentially small. The design method, theoretical characteristics, and experimental results for a 3-dB directional coupler, using double $H-$guide were presented. A well-balanced coupler designed at 94 GHz is achieved, though considerable losses were found. These losses are believed to be caused largely by the teflon's behavior at high frequency. The directivity of the coupler is better than 40 dB. Unfortunately, a broad-band source was not available and consequently no confirmation of the expected 20-GHz theoretical bandwidth was obtained. However, the performance of the coupler at 94.25–95.5 GHz was found to be quite flat. Apart from the insertion loss, the experimental results are in excellent agreement with the theoretical predictions.

Appendix

The differential equation that evolved from the transmission line equations takes the general form

$$\frac{d^2I}{dz^2}+\beta_0^2\left[1+2\frac{\Delta\beta}{\beta_0}f(z)\right]I=0 \quad \text{(A1)}$$

$$\frac{d^2i}{dz^2}+\beta_0^2\left[1-\frac{2\Delta\beta}{\beta_0}f(z)\right]i=0 \quad \text{(A2)}$$

where I and i are the symmetrical and the antisymmetrical normal modes of the coupled system, and $f(z)$ is a function of z that appears because of the variable coupling between the curved transmission lines. In this case, the transmission lines have a parabolic shape and $f(z)$ is given by

$$f(z)=e^{-\alpha z^2} \quad \text{(A3)}$$

where α is equal to hd_0/L^2. The actual coupled fields of the coupled lines are $i_1(z)$ and $i_2(z)$ and given by

$$i_1(z)=\tfrac{1}{2}[I(z)+i(z)]$$
$$i_2(z)=\tfrac{1}{2}[I(z)-i(z)]. \quad \text{(A4)}$$

Let

$$\epsilon=2\frac{\Delta\beta}{\beta_0} \quad \text{(A5)}$$

$$a^2(z)=1+\epsilon f(z) \quad \text{(A6)}$$

where ϵ is small compared to unity since weak coupling is assumed. Upon substituting (A5) and (A6) into (A1) and then setting

$$s=\beta_0\int_{z_0}^{z}a(t)\,dt \quad \text{(A7)}$$

(A1) becomes

$$\frac{d^2I}{ds^2}+\frac{1}{\beta_0}\frac{a'(z)}{a^2(z)}\frac{dI}{ds}+I(z)=0 \quad \text{(A8)}$$

where $a'(z)$ means $(d/dz)a(z)$.

In order to eliminate the middle term of (A8) let

$$I(z)=\frac{U(z)}{\sqrt{a(z)}} \quad \text{(A9)}$$

to get

$$U''(s)+[1-b(s)]U(s)=0 \quad \text{(A10)}$$

with

$$b(s)=\frac{1}{2\beta_0}\frac{d}{ds}\frac{a'(z)}{a^2(z)}+\frac{1}{4\beta_0^2}\frac{a'(z)}{a^2(z)}. \quad \text{(A11)}$$

Assuming a solution of the form

$$U(s)=U_0+\frac{U_1}{\beta_0}+\frac{U_2}{\beta_0^2}+\cdots. \quad \text{(A12)}$$

With $\beta L\gg 1$, and then substituting (A12) into (A10) and equating coefficients of powers of $1/\beta_0$, we obtain the series of equations

$$U_0'' + U_0 = 0 \tag{A13}$$

$$U_1'' + U_1 = \frac{U_0}{2\beta_0}\frac{d}{ds}\left(\frac{a'(z)}{a^2(z)}\right)$$

$$\vdots \tag{A14}$$

The function $U_0(s)$ is determined by the unperturbed equation (A13). The solution of (A13) represents the zero-order solution and is given by

$$U_0(s) = A^{-js} + Be^{+js}. \tag{A15}$$

Substituting (A7) into (A15), then using (A9), we obtain

$$I(z) = \frac{1}{\sqrt{a(z)}}\left[Ae^{-j\beta_0\int_{z_0}^{z} a(t)\,dt} + Be^{+j\beta_0\int_{z_0}^{z} a(t)\,dt}\right] \tag{A16}$$

which is the well-known WKB solution.

Enforcing the boundary conditions of unit amplitude incident wave at $z = -\infty$ and zero reflected wave at $z = +\infty$ (i.e., matched terminations), we get $B = 0$ and

$$A = e^{-j\beta_0\int_{z_0}^{-\infty} a(t)\,dt}. \tag{A17}$$

Therefore, the zero-order solution of (A1) that satisfies the boundary condition is

$$I(z) = \frac{1}{[1+\epsilon f(z)]^{1/4}} e^{-j\beta_0\int_{-\infty}^{z} [1+\epsilon f(t)]1/2\,dt}. \tag{A18}$$

Clearly, the zero-order solution does not include the reflected wave. In order to obtain an expression for the reflected wave, the nonhomogeneous differential equation given by (A14) is solved and the solution obtained is

$$U_1(s) = C_1 e^{-js} + C_2 e^{+js} + \frac{e^{-js}}{4j\beta_0}\int_{r_1}^{s} e^{js'} U_0(s') \frac{d}{ds'}\left(\frac{a'(z')}{a^2(z')}\right) ds' - \frac{e^{+js}}{4j\beta_0}\int_{r_2}^{s} e^{-js'} U_0(s') \frac{d}{ds'}\left(\frac{a'(z')}{a^2(z')}\right) ds' \tag{A19}$$

where C_1 and C_2 are constants yet to be determined by the boundary conditions and r_1 and r_2 are arbitrary constants.

The last term of (A19), which represents the reflected wave, is identical to the correction term that was derived by Bremmer [16]. Upon choosing $r_1 = -\infty$ and $r_2 = +\infty$, and substituting $U_0(s)$ into (A19), and then putting (A19) into (A16), we obtain

$$I(z) = \frac{1}{\sqrt{a(z)}}\left\{C_1 e^{-j\beta_0\int_0^z a(t)\,dt} + C_2 e^{+j\beta_0\int_0^z a(t)\,dt} + \frac{A}{4j\beta_0} e^{-j\beta_0\int_0^z a(t)\,dt}\int_{-\infty}^{z} \frac{d}{dz'}\left(\frac{a'(z')}{a^2(z')}\right) dz' - \frac{A}{4j\beta_0} e^{+j\beta_0\int_0^z a(t)\,dt}\int_{-\infty}^{z} e^{-2j\beta_0\int_0^{z'} a(t)\,dt} \frac{d}{dz'}\left(\frac{a'(z')}{a^2(z')}\right) dz'\right\}. \tag{A20}$$

By changing the lower limits of the integrals that appear in the first two terms to $-\infty$ and $+\infty$, respectively, integrating the double integrals, and then combining similar terms, (A20) can be simplified to

$$I(z) = \frac{1}{\sqrt{a(z)}}\left\{C_1' e^{-j\beta_0 z} e^{-j\Delta\beta\int_{-\infty}^{z} e^{-\alpha t^2}dt} + C_2' e^{j\beta_0 z} e^{j\Delta\beta\int_{\infty}^{z} e^{-\alpha t^2}dt} - \tfrac{1}{2} e^{j\beta_0 z} e^{-j\Delta\beta[\sqrt{\frac{\pi}{4\alpha}} - \int_0^z e^{-\alpha t^2}dt]} \cdot \int_{\infty}^{z} \frac{a'(z')}{a(z')} e^{-2j\beta_0 z'} e^{-j\Delta\beta\int_0^{z'} e^{-\alpha t^2}dt} dz' \right\} \tag{A21}$$

Enforcing the boundary conditions of unit incident wave at $z = -\infty$ and matched terminations at $z = +\infty$, we get $C_1' = 1$ and $C_2' = 0$, and, therefore

$$I(z) = \frac{1}{\sqrt{a(z)}}\left\{e^{-j\beta_0 z} e^{-j\Delta\beta\int_{-\infty}^{z} e^{-\alpha t^2}dt} - \tfrac{1}{2} e^{j\beta_0 z} e^{-j\Delta\beta[\frac{\pi}{4\alpha} - \int_0^z e^{-\alpha t^2}dt]} \cdot \int_{\infty}^{z} \frac{a'(z')}{a(z')} e^{-2j\beta_0 z'} e^{-j\Delta\beta\int_0^{z'} e^{-\alpha t^2}dt} z' \right\} \tag{A22}$$

By replacing ϵ by $-\epsilon$ in (A22) the solution of (A2) can be found. The expression of $i(z)$ will be identical to that of $I(z)$ with ϵ replaced by $-\epsilon$.

Using (A4), the expressions of the actual coupled field amplitudes are evaluated at $z = +\infty$ and found to be, to this order

$$i_1(\infty) = \cos\left[\Delta\beta\sqrt{\frac{\pi}{\alpha}}\right]$$

$$i_2(\infty) = -j\sin\left[\Delta\beta\sqrt{\frac{\pi}{\alpha}}\right]. \tag{A23}$$

At $z = -\infty$ the amplitudes of the reflected wave are given by

$$i_1(-\infty) = -2(\Delta\beta)^2\frac{\pi}{\alpha}e^{-\beta_0^2/\alpha} + 0(\Delta\beta)^4 \tag{A24}$$

and

$$i_2(-\infty) = \Delta\beta\sqrt{\frac{\pi}{\alpha}}\,e^{-\beta_0^2/\alpha} + 0(\Delta\beta)^3 \tag{A25}$$

where α is equal to hd_0/L^2.

Acknowledgment

Many thanks are due to P. Hudson and M. Weidman, of the National Bureau of Standards, for their assistance in the measurements. The authors are also indebted to

Dr. E. F. Kuester for a useful discussion about Bremmer series. In addition, Mr. Abouzahra wishes to thank Mr. S. Al-Marzouk for his support and encouragement.

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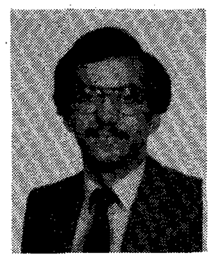

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